Bayesian serial linear regression models for forecasting the short-term abundance of commercial snow crab (*Chionoecetes opilio*)

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ABSTRACT

A Bayesian serial linear regression model is developed to predict recruitment to the fishery in area 12 in the southern Gulf of Saint Lawrence for 2006, 2007 and 2008. The projections are based on estimated abundances of recruitment stages estimated from annual trawl survey data. Under different assumed exploitation rates of $r = (0.35, 0.40, 0.45, 0.50)$, remaining commercial abundances after the fishery and associated quantitative risks as a function of quota level are provided from 2006 to 2009. Effort is made to include various sources of uncertainty, including missing data, diffuse priors and observation errors. Results show a projected decline in fishery recruitment for 2008-2009 with respect to present levels. Cumulative posterior probability plots show the interchange between remaining abundances as well as projected risks under various assumed exploitation rates. All results are presented with associated errors.

RÉSUMÉ

Un modèle hiérarchique Bayesien est développé afin de prédire les niveaux de recrutement dans la zone 12 dans le sud du Golfe du Saint-Laurent pour les années 2006, 2007 et 2008. Les projections sont basées sur des estimées d'abondance de stades de recrutement obtenu à partir d'un relevé au chalut annuel. En partant d'un ensemble de différents taux d'exploitation, soient $r = (0.35, 0.40, 0.45, 0.50)$, on estime l'abondance commerciale restante ainsi que les risques associés à divers niveaux de contingents pour les années 2006 à 2009. On inclus dans le modèle plusieurs sources d'incertitudes incluant des données manquantes, des distributions a priori diffuses ainsi que les erreurs d'estimation. Les résultats démontrent un déclin dans l'abondance du recrutement à la pêcherie pour 2008 et 2009 par rapport aux niveaux présents. Les distributions cumulatives a postériori montrent la relation inverse entre les débarquement projetés et l'abondance de la fraction restante après la pêche sous différent taux d'exploitation. Chaque résultat est présenté avec son propre niveau d'incertitude.

INTRODUCTION

The snow crab (*Chionoecetes opilio*) fishery in the southern Gulf of St. Lawrence (Figure 1) first developed in the late 1960s and has become one of the most lucrative fisheries in Atlantic Canada. The fishing grounds are divided into fishing zones of which area 12 (Figure 1) is the largest, both in terms of fishing area and annual landings. The Department of Fisheries and Oceans (DFO) has conducted a dedicated annual trawl survey (after the fishery, during the months of August and September) since 1988 to provide stock status information. In particular, commercial abundance indices are estimated for the following year and total quotas are usually set as some fraction thereof.

Snow crab grow by a process called *moulting* whereby the animal sheds its old shell and a new, soft shell is secreted and hardens at a larger size (Ruppert & Barnes, 1994). A size-based growth model for snow crab is used to quantify recruitment stages from size-frequency data from the survey. Recruitment stage in this context refers to a group of snow crab which will recruit to the fishery in subsequent years. Only mature males are landed; the females being much smaller are not retained in large-meshed crab traps. For this reason, only male snow crabs are considered in this report.

The objectives of the present article are two-fold. First, a quantitative predictive model of the fisheries recruitment is developed based on estimated recruitment stages for three years into the future. Second, under various hypothesized exploitation rates, we make predictions with regards to the remaining abundance after the fishery as well as the quantitative risks associated under various possible quota levels. Under the Bayesian paradigm, we incorporate several sources of uncertainty including observation and process error. As the level of recruitment is highly variable and seems to follow waves or recruitment pulses, such a model would be a useful tool for fisheries management.

MATERIALS AND METHODS

DATA

The data used are mainly population abundance estimates for zone 12 (Figure 1) derived from the DFO annual snow crab trawl survey in the southern Gulf of Saint-Lawrence for 1988 to 2005. The details of the survey such as gear used and sampling

design are provided by Moriyasu et al. (1998). Abundances were calculated via a geostatistical method called kriging (Conan 1985; Goovaerts 1997; Moriyasu et al. 1998) and mean estimates and standards errors were obtained for all categories. The demographic categories considered are the future recruitment to the fishery, commercial crab remaining after the fishery and commercial crab landed.

Four recruitment stages of male snow crab, which recruit to the fishery in one, two, three and four years are considered. Earlier recruitment categories were not considered as the efficiency of the trawl net gradually decreases for smaller sized individuals and reliable estimates of abundance for these are unavailable. The size limits for these categories were set using a growth model for adolescent male snow crab described by Hébert et al. (2002) as well as other relevant information such as morphometric maturity (Comeau et al. 1998) and minimum legal size (Table 1).

The commercial crab remaining after the fishery is that part of the exploitable resource which was not caught during the fishery. These are legal-sized (carapace width greater than 95 millimeters), morphometrically mature male snow crab with carapace condition 3, 4 or 5. This last criterion is a qualitative measure of relative age of the carapace. Landed commercial crab abundances are calculated from total reported landings and mean weight estimates. Complete nomenclature, classification criteria and descriptions for analytical variables are given in Table 1.

Abundance estimates were transformed to the logarithmic scale prior to the analysis given that they are assumed to be strictly positive and that their associated error increases with the mean. Explicitly, for a given mean abundance estimate μ with associated variance σ^2, a lognormal random variable X was parameterized such that its' first two moments matched μ and σ^2, respectively. It can be shown that if X follows a lognormal distribution parameterized by $\tilde{\mu}$ and $\tilde{\sigma}^2$ ($X \sim LN(\tilde{\mu}, \tilde{\sigma}^2)$) where $\tilde{\sigma}^2 = \ln\left\{\frac{\sigma^2}{\mu^2} + 1\right\}$ and $\tilde{\mu} = \ln \mu - \frac{\tilde{\sigma}^2}{2}$, then X has the two desired properties, namely $E[X] = \mu$ and $Var[X] = \sigma^2$ (Rice, 1995). We also note that if $Z = \ln X$, then $Z \sim N(\tilde{\mu}, \tilde{\sigma}^2)$. These transformations were performed on all abundance data before analysis.

MODEL

The basic model used to relate random variables from one demographic group to another is the log-normal linear model. This model is multiplicative rather than additive and can be expressed as

$$Y = X^a \cdot e^b \cdot e^{\varepsilon}, \qquad \varepsilon \sim N(0, \sigma_p^2) \tag{1}$$

where a, b and σ_p^2 are model parameters and X, Y and ε are random variables. Note that σ_p^2 is referred to as the *process* error on the log-scale in contrast with the errors associated with X and Y, which are referred to as *observation* errors. The model is easily linearized to an additive linear regression model by taking the logarithm on both sides, which yields

$$\ln Y = a \ln X + b + \varepsilon, \qquad \varepsilon \sim N(0, \sigma_p^2) \tag{2}$$

Given the lack of experimental information on the relationship between predictor and response variables the above linear model was assumed because of its simplicity and ubiquitous use as an approximate model.

Fisheries recruitment component

Let $R_i^{(j)}$ be the predicted abundance of a given recruitment class from survey year i that will recruit to the fishery in j years. For example $R_{1989}^{(1)}$ would be the abundance of the recruitment class from the 1989 survey year that will recruit to the fishery in one year (1990). Since the $R_i^{(j)}$'s are not observed directly, we relate them to their kriging estimates, given by their transformed means $\tilde{\mu}_i^{(j)}$ and standard deviations $\tilde{\sigma}_i^{(j)}$ (Table 2).

We relate successive recruitment classes ($R_{i+1}^{(j)}$ and $R_i^{(j-1)}$) using the lognormal linear model: $\ln R_{i+1}^{(j)} = \alpha_j \ln R_i^{(j-1)} + \beta_j + \varepsilon_j$, where α_j and β_j are regression coefficients and $\varepsilon_j \sim N(0, \sigma_j^2)$, where σ_j is the *process* error. Regression coefficients and process errors were all given diffuse priors (Table 4). This model links the recruitment classes in a sequence of lognormal linear regressions: $R_{i+1}^{(1)}$ is conditioned on $R_i^{(2)}$, $R_i^{(2)}$ is conditioned on $R_{i-1}^{(3)}$ and $R_{i-1}^{(3)}$ is conditioned on $R_{i-2}^{(4)}$. To complete the structure, we

specify that $R_i^{(4)}$ be conditioned on itself as part of a first-order autoregressive AR(1) model on the log-scale, where $\ln R_{i+1}^{(4)} = \alpha_4 \ln R_i^{(4)} + \beta_4 + \varepsilon_4$ and α_4 and β_4 are constants and $\varepsilon_4 \sim N\left(0, \sigma_4^2\right)$. A model is specified for the $R^{(4)}$ group to provide a framework for inferring missing data in the series. The complete structure for the recruitment part of the model is presented as a directed acyclic graph (DAG) in Figure 2.

Missing Data

There are two types of missing data, one resulting from the absence of a trawl survey in 1996 and the other resulting from the model structure. Since the above model relates diagonal elements from table 2, we have only partial data at the ends of the time series, in particular the variables of interest $R_i^{(1)}$ for $i = (2006, 2007, 2008)$. Since we wish to use the maximum available information, we also have missing data at the other end of the time series, notably $R_i^{(4)}$ for $i = (1985, 1986, 1987)$. Since almost all missing values $R_i^{(j)}$ are explicitly stated as being part of one model or another, prior distributions need not be stated. The sole exception for this is the first element of the AR(1) model, $R_{1985}^{(4)}$, for which was given a diffuse prior of $\ln R_{1985}^{(4)} \sim N(0, 1000)$. Missing kriging standard deviations were all given informative priors (Table 4) corresponding to a fitted gamma distribution to the set of standard deviation estimates of all available data $\tilde{\sigma}_i^{(j)} \sim Gam(4.6, 37.8)$ (see Appendix A for probability distribution functions and parameterizations).

Remaining abundance and quota risk analysis

The remaining abundance refers to the portion of exploitable part of the total abundance which remains after the fishing season. This portion, added to the one-year recruitment to the fishery $R_i^{(1)}$, provides an index of the total exploitable abundance for the succeeding year and quotas are generally set as a fraction thereof. The goal is to build a predictive model of the remaining abundance for year $i+1$ based on the remaining abundance of year i, the recruitment to the fishery in year i and catches in year $i+1$.

Let E_i be the remaining abundance of mature male snow crab with shell condition 3,4 or 5 for year i and let C_{i+1} be the number (in millions) of individuals caught in year $i+1$. As was done for recruitment abundance variables, estimates for E_i are linked to kriged mean estimates $\mu_i^{(E)}$ and standard errors $\sigma_i^{(E)}$ (Table 3) via their transformed counterparts $\tilde{\mu}_i^{(E)}$ and $\tilde{\sigma}_i^{(E)}$ on the log-scale. Catch numbers C_{i+1} are estimated from total landings L_{i+1} (Table 3) registered for zone 12 and mean weights w_{i+1}, estimated from port samples measured and transformed using a size-weight relationship. The annual mean weights were assumed to follow a normal distribution ($w_i \sim N\left(\mu_w, \sigma_w^2\right)$) with diffuse priors $\mu_w \sim N\left(0, \left(10^3\right)^2\right)$ and $\sigma_w^{-2} \sim Gam\left(10^{-4}, 10^{-4}\right)$.

We relate E_{i+1} to E_i, R_i and $\ln C_{i+1}$ using the lognormal linear model $\ln E_{i+1} = \kappa\left(\ln R_i^{(1)} + \ln E_i\right) - \ln C_{i+1} + \varepsilon_E$, where $\varepsilon_E \sim N\left(0, \sigma_E^2\right)$ and κ is a linear regression coefficient. Diffuse priors were provided for the regression coefficient κ and the process error σ_E^2 (Table 4). Heuristically, this model seeks to account for the discrepancy between the predicted total fishable abundance (composed of fisheries recruitment and remaining abundance) and the catches and remaining abundance for the following year. The model in DAG form is presented in Figure 3.

Under an assumed model, one may assess the risk associated with various future quota levels, which is of particular interest to stakeholders and fisheries managers. More precisely, we calculated the probability that a target exploitation rate r would be exceeded for a series of hypothetical quota levels ranging from 0 to 50000 MT for each year from 2006 to 2009 and each of four target exploitation rates $r = (0.35, 0.40, 0.45, 0.50)$. These quota levels, defined in 1000MT increments by $L'_j = 1000\,j$, for $j = 0, \ldots, 50$, were combined with a randomly-generated mean weight value w_{i+1} from its posterior distribution $w_i \sim N\left(\mu_w, \sigma_w^2\right)$ (see subsection below) to obtain stochastic catch abundances values $C'_{i+1,j}$, where $i = 2005, \ldots, 2008$, for each quota level. We then combined the quota-specific catch abundance $C'_{i+1,j}$ with estimated and projected remaining E_i and recruitment abundances $R_i^{(1)}$ to obtain a simulated

exploitation rate, calculated as $r'_{i+1,j} = C'_{i+1,j} / \left(E_i + R_i^{(1)}\right)$. The probability that this value exceeds the target exploitation rate r was then estimated as $\pi_{i+1,j} = P\left[r'_{i+1,j} > r\right]$ via simulated values. The DAG for these calculations is presented in figure 4. These results are presented numerically in Table 8 as quota levels for given odds ratios 1:4, 1:1 and 4:1 which represent the risk of exceeding the target exploitation rate r. Figure 7 shows the level of risk as a continuous function of quota level for year 2006 to 2009 for all four assumed exploitation levels.

Model fitting and parameter inference

The model is formulated under the Bayesian paradigm (Bernier et al., 2000; Congdon, 2001; Gelman et al. 2004), which requires prior probability models for the parameters as well as the data model. Inferences are based on what are called posterior distributions of the parameters, which arise from synthesis of our prior assumptions on the model parameters and the likelihood of the data given the analytical model. The posterior distribution follows naturally from Bayes' theorem as the distribution of the parameters, given the data. In practice, posterior inference is difficult as the functional form of the posterior distribution is too complex for closed-form analytical treatment, except for certain model classes. However, general methods are available which allow the analyst to generate random samples from the posterior distributions without having to fully derive their functional forms. These methods, such as rejection sampling, importance sampling, Gibbs sampling and Metropolis-Hasting sampling are collectively referred to as Monte Carlo (MC) methods while the latter two, as they generate Markov chain sequences are referred to as Markov Chain Monte Carlo (MCMC) methods (Congdon, 2001; Gelman et al. 2004).

Since our "data" are in fact parameter estimates from a prior analysis, we incorporate both the estimates and their errors (treated as observation errors) into the regression model. Missing data, which are generally problematic, are treated as unknowns within the model. Inferences on the model parameters and random variables are made via simulation of posterior distributions by Markov Chain Monte Carlo (MCMC) sampling using WinBUGS (Gilks et al. 1994). A single chain of one hundred thousand samples were generated for posterior inference after a burn-in period of 5000

samples (initial samples are rejected as a precaution as the MCMC sequence may not have converged). The analysis was repeated a number of times to insure that they converged to the same solution each time. The program, available upon request from the authors, is also listed in Appendix B.

RESULTS

MCMC sample chains generally showed good mixing (low autocorrelation between samples) and did not exhibit non-stationary behavior, meaning that no trends were apparent within the ordered samples as they were generated. Different initial parameter values converged to the same posterior distributions. On rare occasions, every 50000 samples or so, numerical overflow errors appeared as extreme values when predicting catch abundances and landings for 2006 to 2009. This was due to accumulating errors in the right-hand tails of log-normal distributions which tend to be very long. A cut-off point of approximately one billion ($e^7 \cdot 10^6$) for predicted catch abundances, considered to be a conservative value, was imposed on the chains to control this behavior. Imposing this constraint did not appear to have any significant effect on the results presented, as setting it to different levels (e.g. $e^6 \cdot 10^6$ or $e^8 \cdot 10^6$) did not result in any apparent change.

Posterior summary statistics for model parameters are shown in Table 5. As is corroborated by comparing the means and median values, most distributions were symmetrically distributed, with the exception of the error parameters which were right-skewed. The regression coefficients α_j were generally between zero and one, tending towards the latter. The β_j's had rather wide distributions and included zero for $j = 1,2,3$, indicating that they may not play a significant role in the overall fit of the model. As expected, the α_js and β_j are correlated for a given j, as is generally observed in the simple linear case. There are trends within the α_j and β_j's which decrease and increase respectively, as a function of recruitment stage $j = 1,2,3,4$. This shows a weakening proportional relationship between earlier adjacent recruitment stages. The error

parameters σ_1, σ_2 and σ_3 have smaller values than either σ_4 (in the log-scale AR(1) model) or σ_E (remaining abundance model). The larger the error the less precise predictions based on these values will be. The mean weight parameters were 560.9 grams for μ_w with a standard deviation parameter of σ_w = 25.6 grams.

Posterior MCMC sample statistics for recruitment abundance are presented in Table 6. The values for all results preceding 1988, exceeding 2005 or from 1996 (this year of the missing survey) are shown. The complete data series are shown for each recruitment stage (1 through 4) as box-plots in Figure 5. The wave-pulse behavior of the recruitment is quite apparent.

The levels of one-year recruitment $R_i^{(1)}$ are predicted to drop considerably in 2006, 2007 and 2008; the levels in 2008 are expected to be less than half those in 2005 and those predicted for 2006. Of course uncertainty about these values must be considered, though the decrease is observed for the latter years in all four recruitment stages.

The posterior predicted remaining abundances E_i for $i = 2006,\ldots,2009$ under various exploitation rates r are presented in Table 7. Estimates for 2007, 2008 and 2009 were highly skewed, owing to their log-normality and relatively large variance, bringing about differences between their mean and median values. These results are presented graphically in Figure 6, which shows that a lower exploitation rate yields larger remaining abundances through time, although the predictive errors are fairly large.

The risk that the quota-specific simulated exploitation rate $r'_{i+1,j}$ exceeds the target exploitation rate r for years 2006-2009 is presented numerically in Table 8 and graphically in Figure 7. Risk probabilities in Table 8 are given as odds ratios of 1:4, 1:1 and 4:1 for each target exploitation level for each year. What is given are the quota levels at which the associated level of risk are met. For instance, the quota level at which there are 1:4 odds that the target exploitation level of r = 0.40 is exceeded for 2007 is 17870 MT. Figure 7 reads similarly, but the quota levels and risk probabilities are plotted continuously for all levels up to 50000 MT.

DISCUSSION

Recruitment to the Fishery

The recruitment model is structured in a similar manner as a population model, with each recruitment stage expressed as a function (linear function on the log-scale) of the preceding stage from the year before. However, the model parameters must not be interpreted as having a simple meaning directly related to specific population dynamic processes. In fact, the identification of recruitment stages from survey data is complicated by the stochastic nature of the growth process, which does not allow us to classify individuals by simple size intervals as we have done here. Our approach is therefore an approximation. Furthermore, there are other processes such as individuals who forego their moult (called skip-moulters) and temporal variability in moult to maturity probabilities, as well as reduced catchability within the sampling gear as individual size decreases, all of which further blurs the definition and interactions between successive recruitment groups. We avoided these complications by focusing rather on the *prediction* of recruitment by a linear model, which is understood to be an approximation and thus avoided making specific interpretations of its parameters as they relate to specific processes within the population.

Certain regression parameters, notably intercepts, had posterior distributions which included zero, suggesting that our model might be subject to further simplification. Given that the focus of the analysis is in prediction rather than parameter inference and that the former is not adversely affected by this slight overparameterization, the present version of the model was left unmodified.

We note from kriged estimates of recruitment abundances (Table 2 & Figure 5) that recent trends in $R^{(4)}, R^{(3)}, R^{(2)}$ and $R^{(1)}$ are all on the decrease, providing strong evidence that fisheries recruitment in the following years will be on the decline. It is thus probable that the crest of the last recruitment wave has passed and that we are entering a trough in fisheries recruitment. Given the simple structure and accumulating errors, we aren't able to make any inferences as to long-term recruitment patterns.

We note that predictions for $R_i^{(1)}$ for $i = 2006, 2007, 2008$ are based on less and less 'hard' data as time progresses. In fact, $R_{2008}^{(1)}$ is based solely on $R_{2005}^{(4)}$ passed through the chain of fitted hierarchical regressions. On the log-scale (not shown), this translates into increasing error on $\ln R_{2006}^{(1)}$, $\ln R_{2007}^{(1)}$ and $\ln R_{2008}^{(1)}$, but at the absolute scale, this effect is attenuated by the downward trend of the data, which has a deflating effect on the predicted variances through time. This is a consequence of the assumption of log-normally distributed errors, which are correlated with the mean.

Being able to treat 1996 missing data as part of the model has enabled us to maximally use all available data. To treat only complete data in each sub-model would have resulted in a loss of information. Although not a primary objective, this approach allows us to make useful inferences regarding abundance estimates for 1996.

Since some effort was made to include various sources of uncertainty in the model, it would be wise to make use of this when making inferences. In particular, credibility intervals (Table 7) and interquartile distances are useful in assessing the error associated with each predicted recruitment estimate.

Residual plots for the $\ln R^{(1)}$, $\ln R^{(2)}$ or $\ln R^{(3)}$ recruitment variables versus their predicted values showed little evidence of either auto-correlation or variance heterogeneity. The AR(1) model for the $\ln R^{(4)}$ series, on the other hand, had auto-correlation present, implying that the degree or type of the AR model may not be adequate. However, as the prediction of future values for the other recruitment stages do not depend on predicted values of $R^{(4)}$'s, model misspecification does not pose a serious problem. Predicted values of $R^{(4)}$'s for 1985, 1986, 1987 and 1996 thus probably have inflated errors and some care must be taken when interpreting these estimates.

Remaining abundance and risk analysis

Predicted remaining abundances (Table 7 & Figure 6) show similar values for 2006 and 2007 and then declines in 2008 and 2009. The amount of remaining abundance is higher when exploitation rates are lower. Higher levels of remaining abundance may be desirable for biological reasons, such as improved mating success, or to act as a buffer during periods of low fisheries recruitment. High exploitation rates make the fishery

more dependent on recruitment input, and thus more subject to its fluctuations. The error about these predicted values is however very high, suggesting a poor relationship between remaining and catch abundances in the regression model. Log-scale residuals also showed some autocorrelation between the errors. This may suggest that some other, unaccounted process may be at work, contributing to high prediction errors.

As the level of expected fisheries recruitment decreases, so too do the expected quota levels for a given exploitation rate and fixed level of risk (Figure 7). The quota levels for a 1:1 risk level show a sharp decline from 2006 to 2009 which is further exacerbated as the exploitation rate is increased. For instance, the decrease from 2006 to 2009 passes from 40% under a 0.35 exploitation rate, to 53% under a 0.50 exploitation rate.

The difficult choice of selecting the appropriate exploitation rate is complex and depends on the particular objectives of a fisheries management plan. The results from the proposed model provide information with respect to a specific aspect of the population dynamics, namely the fisheries recruitment, the remaining abundance and associated risks of various quota levels under various assumed exploitation rates. Other important factors such as natural mortality and the incidence of soft-crab in landings must also be considered by the manager.

There is added uncertainty due to the inconstancy of trawl survey protocols through time. Earlier years generally had fewer survey stations, and improvements in positioning technology (advent of GPS), the use of three different boats, and trawl monitoring equipment have probably had some effect in improving the precision of abundance estimates. However, this adds some uncertainty with regards to the homogeneity of time series data. This being said, the trawl survey protocol has not changed significantly since 1998, and there is no reason to believe that recent trends in future recruitment are due to sampling artifacts, and these trends must be treated as the most reliable indicator available.

In summation, these results suggest a serious decline in fisheries recruitment from 2006-2008, though statistical uncertainties, both data and model-based, must be taken into account. Remaining abundances show an inverse relationship with exploitation rate.

In light of these points, the exploitation rate must be selected with due care the coming years while giving proper consideration to the uncertainties involved.

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Table 1: Variable names, descriptions, units and classification criteria used. ('cw' is carapace width).

Variable	Description	Units	Classification Criteria
$R_i^{(1)}$	Recruitment abundance to the fishery in one year.	$\times 10^6$	Morphometrically mature males, $cw \geq 95mm$, shell condition 1 & 2 (recently moulted)
$R_i^{(2)}$	Recruitment abundance to the fishery in two years.	$\times 10^6$	Morphometrically non-mature males, $cw \geq 83mm$.
$R_i^{(3)}$	Recruitment abundance to the fishery in three years.	$\times 10^6$	Morphometrically non-mature males, $69mm \leq cw < 83mm$.
$R_i^{(4)}$	Recruitment abundance to the fishery in four years.	$\times 10^6$	Morphometrically non-mature males, $56mm \leq cw < 69mm$.
E_i	Remaining fishable abundance after the fishery.	$\times 10^6$	Morphometrically mature males, $cw \geq 95mm$, shell condition 3,4 & 5 (older carapace).
C_i	Catch abundance.	$\times 10^6$	Morphometrically mature males, $cw \geq 95mm$, shell condition 3,4 & 5 (older carapace).
L_i	Landings.	Metric tons	
w_i	Mean weight per year.	grams	

Table 2: Mean (standard deviation) abundances (millions) by recruitment stage obtained from kriging for the survey years, 1988 to 2005. The number in superscript refers to the number of years to recruitment to the fishery. n/a means the data are unavailable for that year.

Survey Year (i)	$\mu_i^{(4)}(\sigma_i^{(4)})$	$\mu_i^{(3)}(\sigma_i^{(3)})$	$\mu_i^{(2)}(\sigma_i^{(2)})$	$\mu_i^{(1)}(\sigma_i^{(1)})$
1988	77.9 (39.3)	34.1 (5.5)	17.8 (4.4)	1.6 (0.6)
1989	143.0 (18.8)	100.0 (10.0)	54.8 (11.8)	7.3 (2.6)
1990	250.0 (20.7)	198.0 (17.1)	122.7 (15.9)	43.9 (8.1)
1991	117.6 (17.9)	116.1 (14.0)	146.6 (16.0)	42.1 (8.7)
1992	69.1 (10.9)	124.0 (16.7)	173.9 (25.5)	72.7 (9.0)
1993	28.4 (5.4)	97.6 (14.9)	161.1 (20.8)	129.7 (20.9)
1994	45.1 (5.5)	101.0 (11.3)	149.1 (12.7)	92.3 (7.5)
1995	42.5 (5.0)	38.6 (5.2)	83.5 (9.6)	53.6 (44.7)
1996	n/a	n/a	n/a	n/a
1997	90.5 (9.1)	54.5 (6.4)	47.7 (5.3)	41.9 (3.6)
1998	111.0 (8.1)	66.5 (5.4)	46.2 (6.8)	35.0 (4.4)
1999	136.0 (15.0)	108.0 (10.5)	96.5 (13.3)	35.5 (3.2)
2000	144.0 (14.1)	107.0 (9.1)	67.6 (6.4)	43.2 (3.7)
2001	221.0 (13.6)	161.0 (10)	91.7 (9.8)	48.4 (5.7)
2002	122.0 (9.8)	163.0 (13.8)	149.5 (12.5)	63.1 (5.1)
2003	82.9 (11.0)	148.0 (10.8)	144.0 (11.1)	62.9 (4.3)
2004	49.5 (5.4)	78.8 (6.3)	134.1 (8.3)	101.0 (4.7)
2005	27.1 (3.6)	54.8 (3.2)	120.8 (8.8)	69.6 (3.6)

Table 3: Mean (standard deviation) adundance (millions) of post-fishery remaining abundance ($\mu_i^{(E)}(\sigma_i^{(E)})$) obtained from kriging, annual fishery landings (L_i ; metric tons) and mein weight of individual crab in the fishery ($\mu_i^{(w)}$; g) for 1988 to 2005. n/a means the data are unavailable for that year.

Survey Year (i)	$\mu_i^{(E)}(\sigma_i^{(E)})$	L_i (MT)	$\mu_i^{(w)}$
1988	15.5 (1.9)	12355	589.4
1989	27.2 (2.4)	7882	576.4
1990	28.2 (2.8)	6950	517.2
1991	35.5 (2.9)	10019	542.7
1992	47.3 (7.4)	11235	559.3
1993	91.1 (7.9)	14336	563.3
1994	106.8 (6.6)	19995	560.9
1995	95.7 (6.4)	19944	587.7
1996	n/a	15978	n/a
1997	37.3 (3.3)	15413	594.2
1998	37.5 (3.6)	11136	586.5
1999	36.4 (3.7)	12682	541.6
2000	8.9 (2.1)	15046	571.7
2001	19.6 (2.4)	13819	524.9
2002	16.3 (1.4)	21869	524.5
2003	34.5 (3.1)	16898	548.6
2004	27.2 (2.4)	26213	565.4
2005	30.2 (2.7)	32400	582.8

Table 4: Summary of prior distributions used in the model (see Appendix for parameterizations and probability distribution functions).

Parameter	Prior Distribution
α_j for $j = 1,\ldots,4$	$\alpha_j \sim N(0, 10^4)$
β_j for $j = 1,\ldots,4$	$\beta_j \sim N(0, 10^4)$
κ	$\kappa \sim N(0, 10^4)$
σ_j for $j = 1,\ldots,4$	$\sigma_j^{-2} \sim Gam(10^{-4}, 10^{-4})$
σ_{rem}	$\sigma_{rem}^{-2} \sim Gam(10^{-4}, 10^{-4})$
$\tilde{\sigma}_i^{(j)}$ (missing data)	$(\tilde{\sigma}_i^{(j)})^{-2} \sim Gam(4.6, 37.8)$
$R_{1985}^{(4)}$	$R_{1985}^{(4)} \sim LN(0, \sigma_4^2)$
μ_w	$\mu_w \sim N(0, 10^6)$
σ_w	$\sigma_w^{-2} \sim Gam(10^{-4}, 10^{-4})$

Table 5: MCMC sample statistics for the posterior distribution of the model parameters CI represents the credibility interval.

Parameter	Mean	Standard Deviation	Lower CI (2.5%)	Median (50%)	Upper CI (97.5%)
α_1	0.91	0.12	0.69	0.91	1.17
α_2	0.84	0.14	0.60	0.83	1.14
α_3	0.79	0.13	0.55	0.78	1.07
α_4	0.54	0.11	0.32	0.54	0.75
β_1	-0.15	0.56	-1.34	-0.11	0.87
β_2	0.77	0.64	-0.61	0.82	1.89
β_3	0.99	0.60	-0.32	1.03	2.06
β_4	2.05	0.48	1.13	2.04	3.02
κ	0.92	0.03	0.87	0.92	0.98
σ_1	0.18	0.07	0.06	0.17	0.34
σ_2	0.25	0.07	0.14	0.24	0.40
σ_3	0.31	0.08	0.19	0.30	0.50
σ_4	0.52	0.10	0.37	0.51	0.76
σ_E	0.82	0.19	0.54	0.80	1.26
μ_w	560.9	6.3	548.2	560.9	573.3
σ_w	25.64	4.86	18.21	24.95	37.11

Table 6: MCMC sampling statistics for the posterior distribution of the predicted values by recruitment stage ($R_i^{(j)}$). CI represents the credibility interval.

Parameter	Mean	Standard Deviation	Lower CI (2.5%)	Median (50%)	Upper CI (97.5%)
$R_{1985}^{(4)}$	0.99	0.13	0.73	0.99	1.277
$R_{1986}^{(4)}$	8.09	4.12	2.79	7.24	18.32
$R_{1987}^{(4)}$	30.14	11.77	12.92	28.26	58.05
$R_{1996}^{(4)}$	54.63	18.32	27.01	51.96	98.33
$R_{1986}^{(3)}$	2.80	1.74	0.57	2.43	7.188
$R_{1987}^{(3)}$	12.75	5.41	4.62	11.96	25.53
$R_{1996}^{(3)}$	46.70	12.08	26.31	45.60	73.47
$R_{2006}^{(3)}$	40.42	15.87	16.94	38.07	77.93
$R_{1987}^{(2)}$	4.67	2.34	1.37	4.26	10.38
R_{1996}^{2}	61.53	11.84	39.96	61.03	86.62
$R_{2006}^{(2)}$	66.10	19.05	35.42	64.02	110.05
$R_{2007}^{(2)}$	51.11	22.02	20.41	47.35	104.11
$R_{1996}^{(1)}$	49.46	10.97	30.34	48.61	73.68
$R_{2006}^{(1)}$	68.83	15.09	43.84	67.25	103.70
$R_{2007}^{(1)}$	40.57	13.47	19.68	38.85	71.79
$R_{2008}^{(1)}$	32.05	14.04	12.87	29.47	66.92

Table 7: MCMC sample statistics of the posterior distribution of the predicted remaining abundances (E_i) for four levels of fixed exploitation rates (r) over four year 2006 to 2009. CI represents the credibility interval.

r	Year (i)	Mean	Lower CI (2.5%)	Median (50%)	Upper CI (97.5%)
0.35	2006	49.77	5.84	33.15	189.65
	2007	55.49	3.79	33.09	241.25
	2008	44.09	2.8	26.475	189.3
	2009	35.43	2.19	21.28	151.5
0.40	2006	43.25	5.07	29.2	166.2
	2007	45.43	3.06	26.935	198.9
	2008	35.56	2.16	21.28	154.85
	2009	28.32	1.67	16.89	123.6
0.45	2006	38.8	4.65	25.98	149.7
	2007	37.97	2.49	22.185	165.8
	2008	28.94	1.66	17.03	126.4
	2009	22.87	1.23	13.35	100.65
0.50	2006	34.64	4.1	23.34	134.05
	2007	32.18	2.07	18.67	140.8
	2008	24.43	1.33	13.97	110.6
	2009	18.72	0.93	10.8	84.28

Table 8: Quota levels by year for odds ratios 4:1, 1:1 and 1:4, which represent the risk of exceeding the target exploitation rate r.

r	Year (i)	4:1	1:1	4:1
0.35	2006	18537	19660	20800
	2007	16143	20387	27446
	2008	10284	14807	23096
	2009	7913	11831	18746
0.40	2006	21248	22500	23779
	2007	17870	22312	29539
	2008	11088	15594	23602
	2009	8306	12235	18981
0.45	2006	23922	25290	26724
	2007	19635	24339	31811
	2008	11789	16325	24039
	2009	8685	12773	19395
0.50	2006	26554	28100	29691
	2007	21310	26174	33640
	2008	12598	17243	24835
	2009	9137	13206	19668

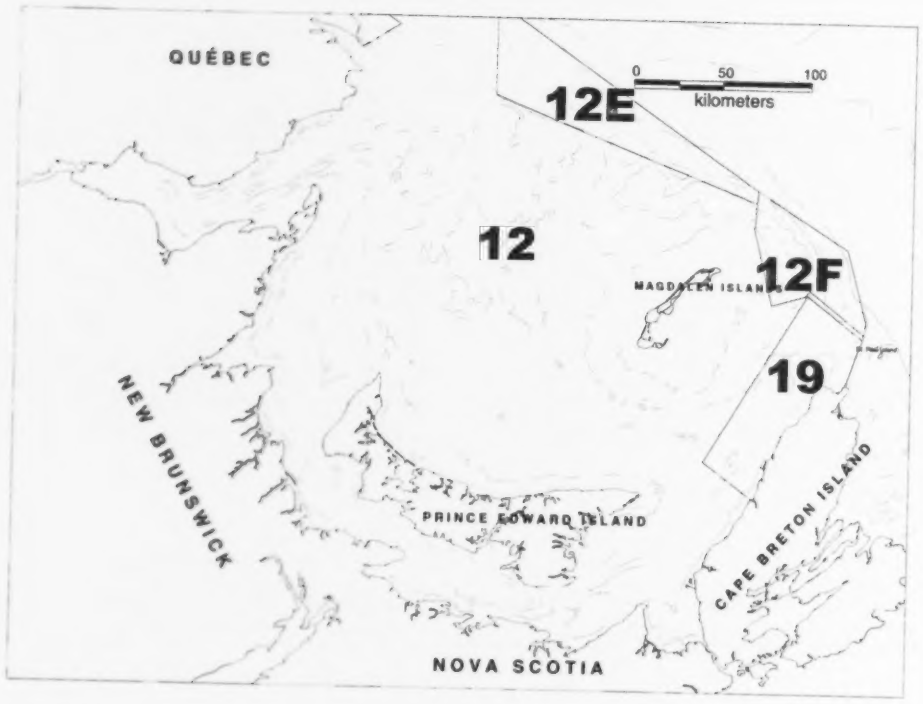

Figure 1: Location of the area 12 fishing zone in the southern Gulf of Saint Lawrence. Areas 12E and 12F are not considered in the present study.

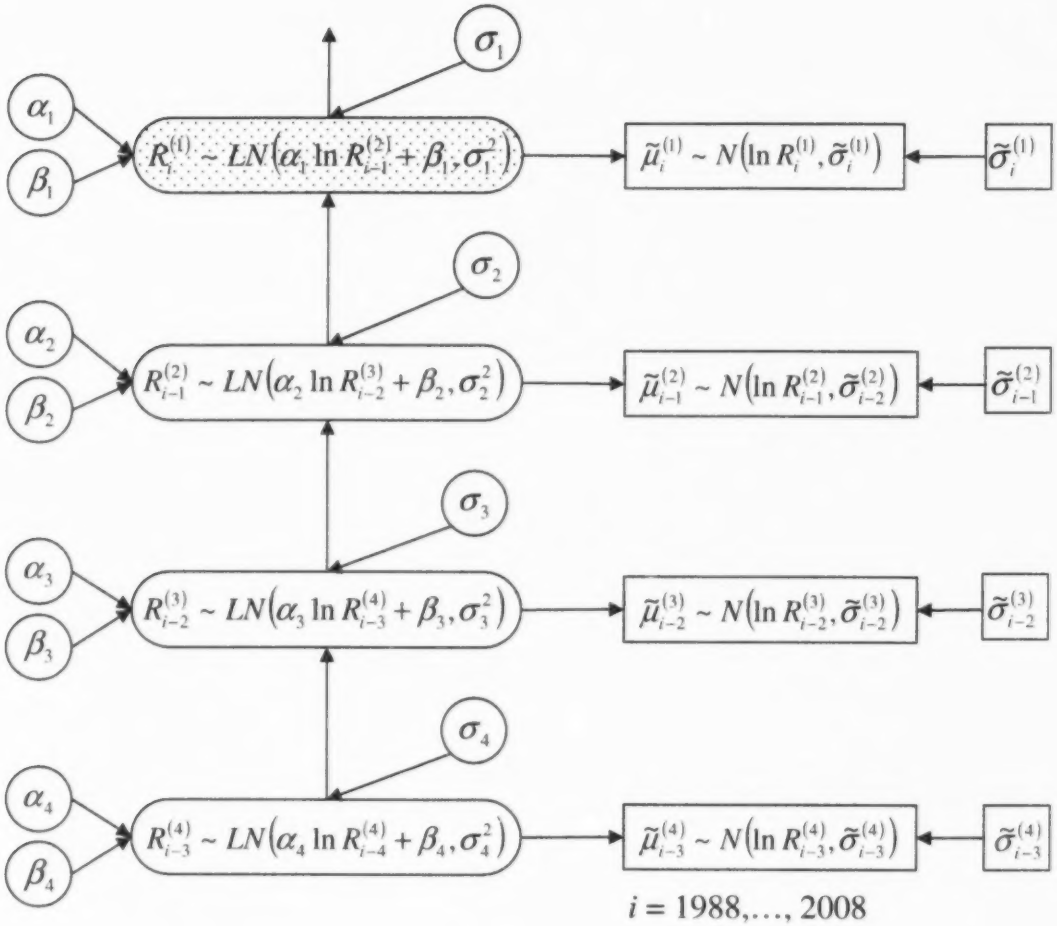

Figure 2: Directed acyclic graph (DAG) of the recruitment part of the model. The shaded node (upper-left), of principal interest, is the predicted recruitment to the fishery in one year and serves as input into the second part of the model (Figure 3).

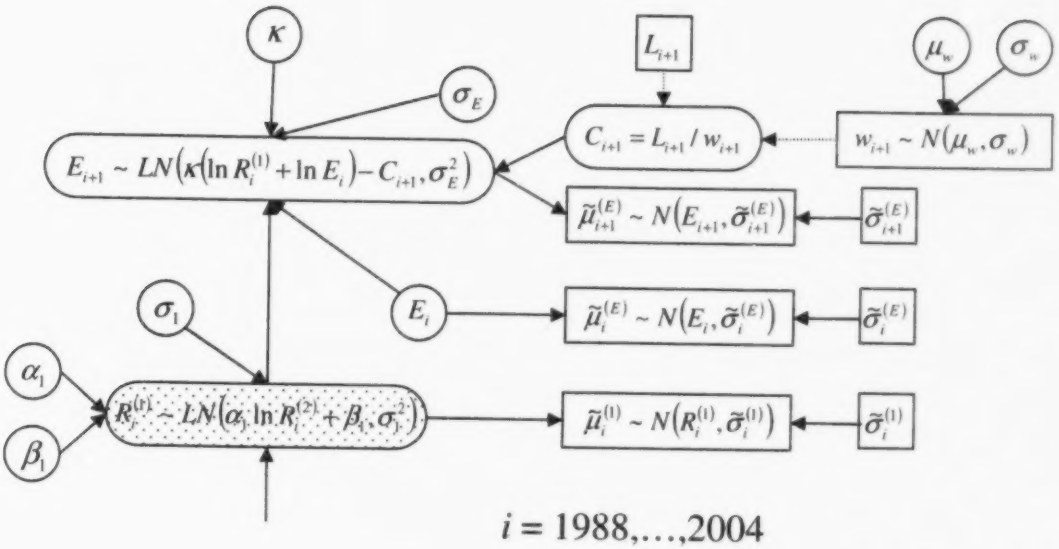

Figure 3: Directed acyclic graph (DAG) of the remaining abundance and catches part of the model for years $i = 1988,\ldots,2004$. Note that the shaded (lower-left) recruitment node is part of the recruitment model (Figure 2).

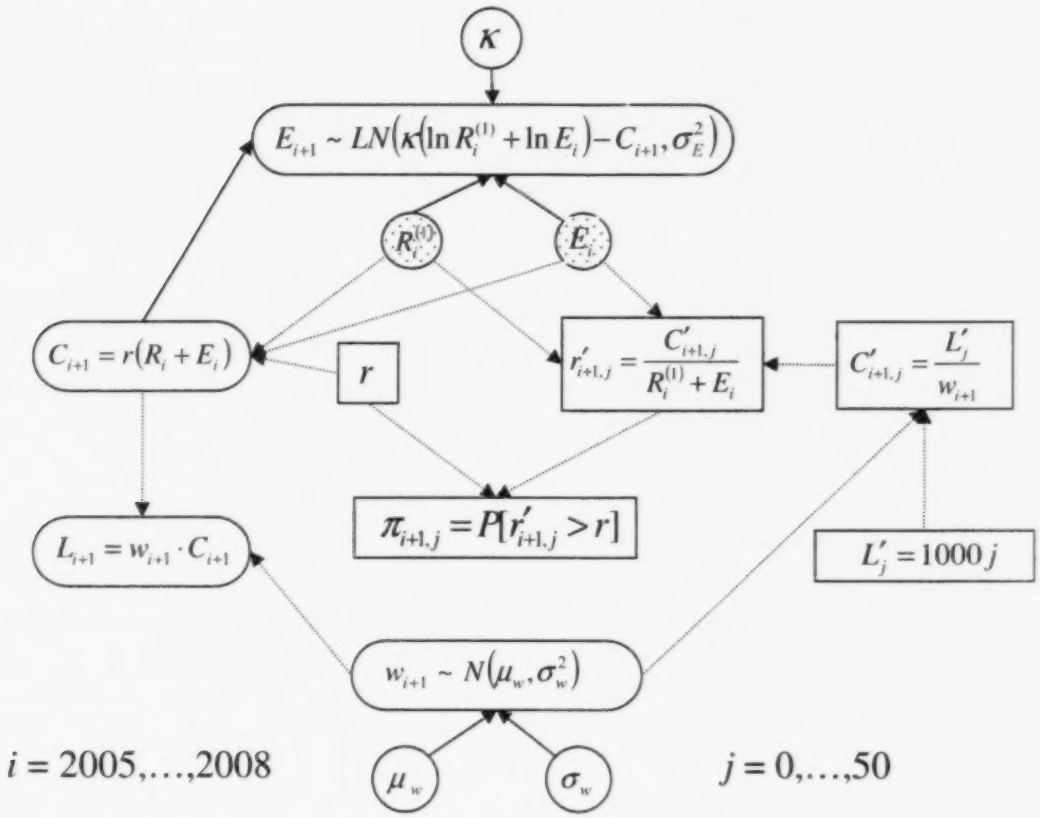

Figure 4: Directed acyclic graph (DAG) for the prediction of remaining abundances as well as the risk of exceeding the target exploitation rate r under varying quota levels, which are indexed by j. This model is connected to past recruitment and remaining abundances through the shaded nodes.

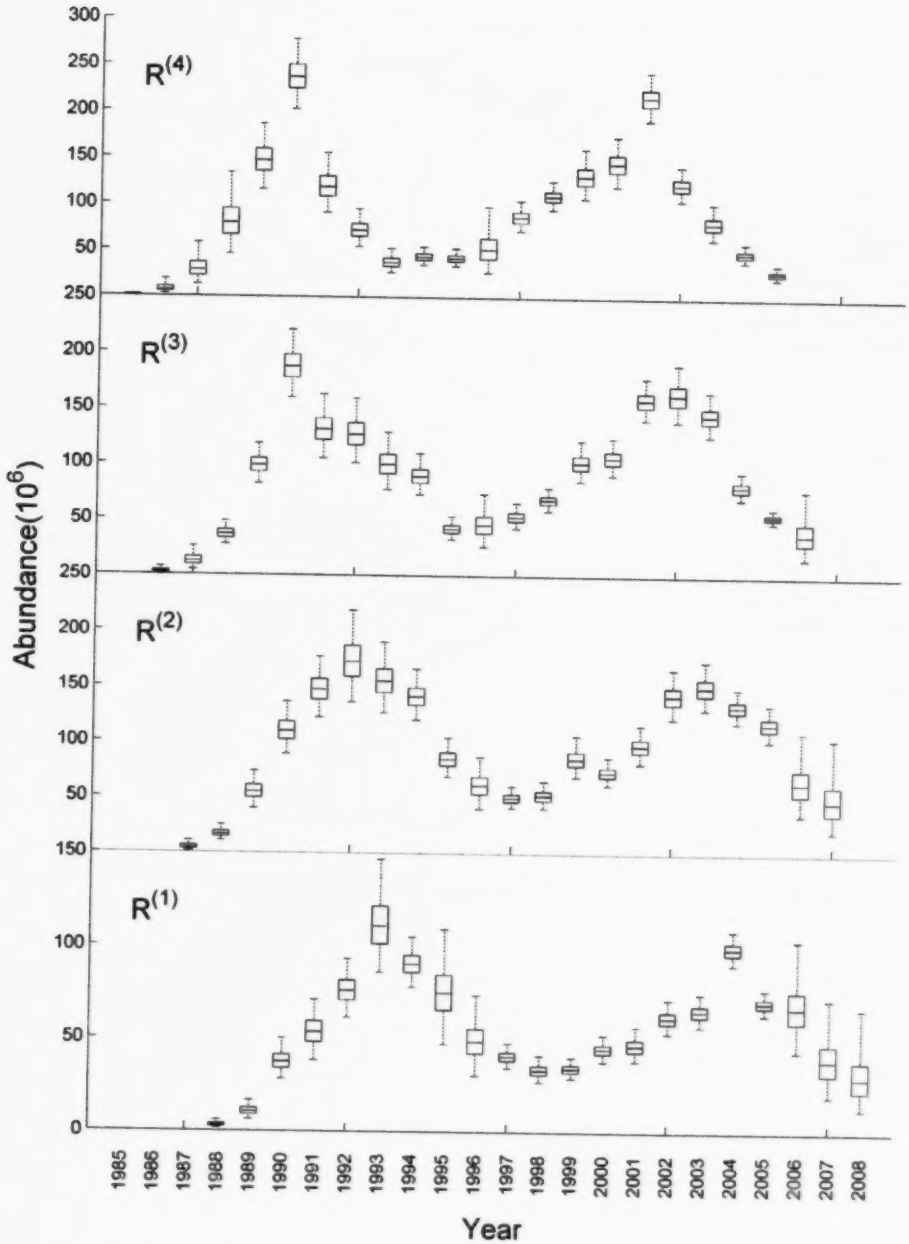

Figure 5: Box plots of recruitment abundance for $R^{(j)}$, where $j = 1,\ldots,4$ year until recruitment to the fishery. The middle line shows the posterior median, the boxes show the 1st (lower bound) and 3rd (upper bound) quartiles, and the whiskers show the 95% credibility interval (2.5% and 97.5% percentile values). Shaded boxes stem directly from data estimates while white boxes are inferred from the model.

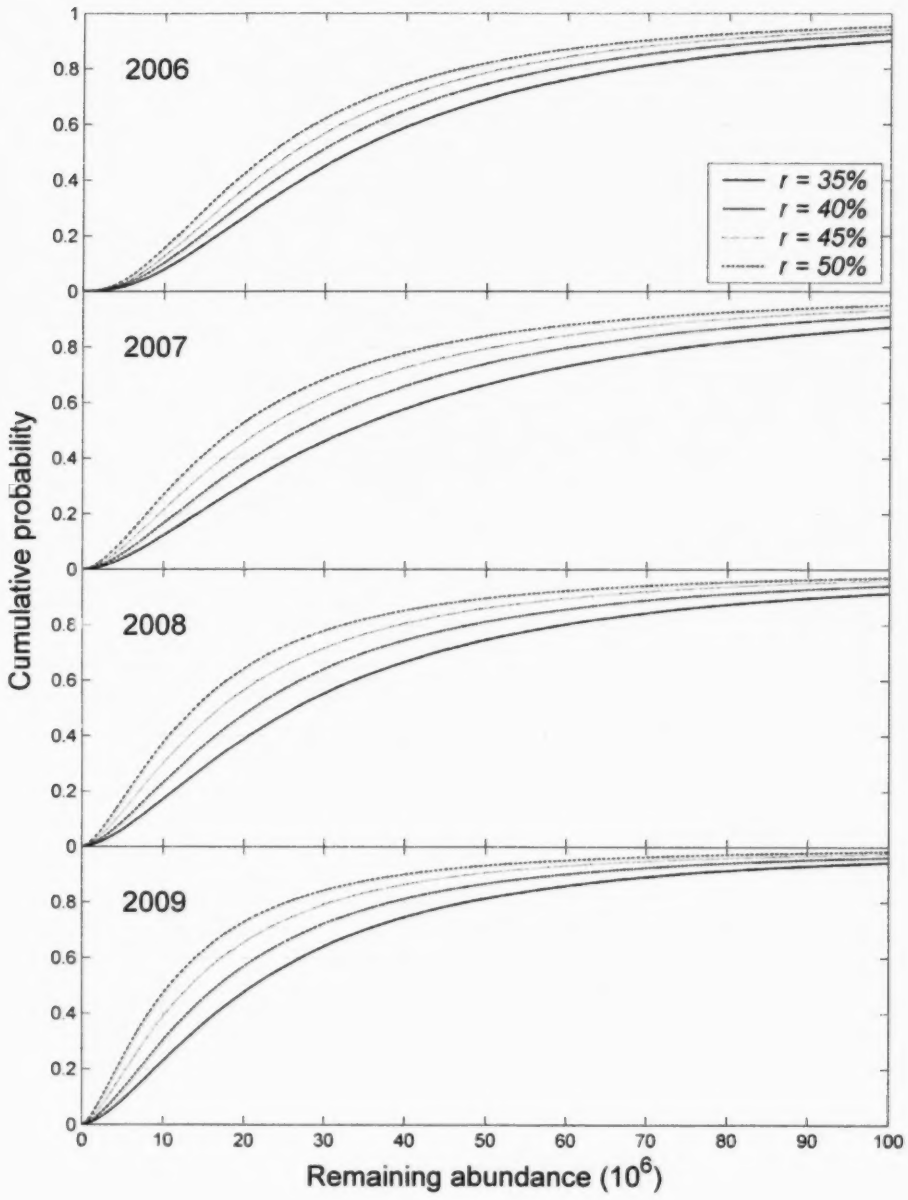

Figure 6: Posterior MCMC cumulative densities for predicted remaining abundances E_i for $i = 2006,\ldots,2009$ under varying exploitation rates (r).

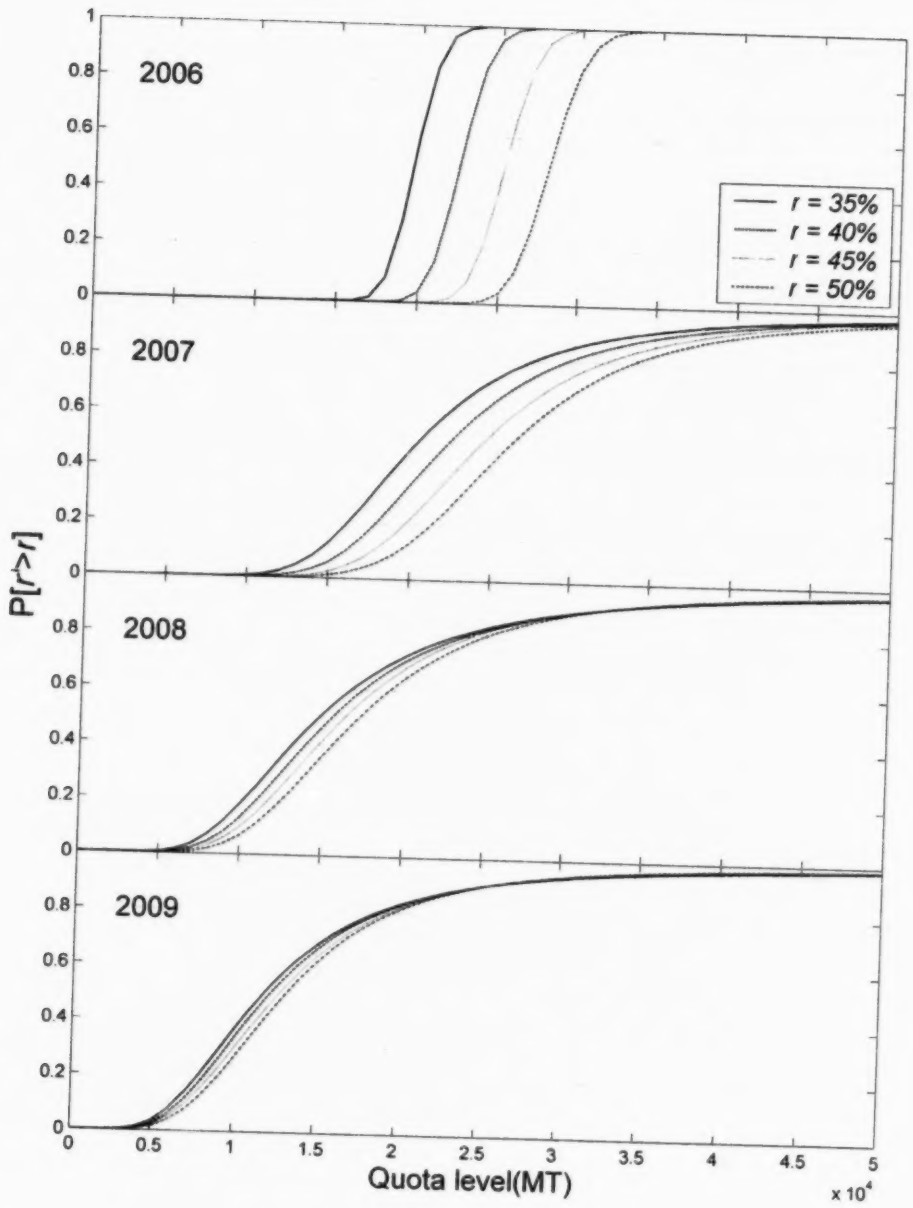

Figure 7: Posterior risk of exceeding target exploitation rate r as a function of quota level for years 2006 to 2009.

APPENDIX A

Probability distribution functions used and their parameterizations

Normal distribution $X \sim N(\mu, \sigma^2)$

(A.1) $$f(x \mid \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{(x-\mu)^2}{2\sigma^2} \right\}$$

Lognormal distribution $X \sim LN(\mu, \sigma^2)$

(A.2) $$f(x \mid \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \cdot \frac{1}{x} \cdot \exp\left\{ -\frac{(\ln x - \mu)^2}{2\sigma^2} \right\}$$

Gamma distribution $X \sim Gam(\alpha, \beta)$

(A.3) $$f(x \mid \alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$

APPENDIX B

WinBUGS code for the described model.

```
# Define model:
model{
        # Provide process parameters prior distributions:
        for  (i in 1:4){
                alpha[i] ~ dnorm(0, 1.0E-4)  # Array of slope parameters.
                beta[i]  ~ dnorm(0, 1.0E-4)  # Array of intercept parameters.
                # Array of process precision parameters:
                tau.process.r[i] ~ dgamma(1.0E-4, 1.0E-4) I(0.000001,)
                sigma.process.r[i] <- 1/sqrt(tau.process.r[i])
        }

        # Provide informative priors for missing kriging errors:
        for (i in 1:4){
                sigma.r[i+8,i] ~ dgamma(4.6, 37.8)
                for (j in (i+1):4){
                        sigma.r[i,j] ~ dgamma(4.6, 37.8)
                }
                for (j in 1:(i-1)){
                        sigma.r[i+17,j] ~ dgamma(4.6, 37.8)
                }
        }

        # Calculate kriging precisions from standard errors:
        for (i in 1:N) {
                for (j in 1:4){
                        tau.r[i,j] <- 1/pow(sigma.r[i,j],2);
                }
        }

        # State autoregressive AR(1) model for R4:
        log.R[1,4] ~ dnorm(0,tau.r[1,4])
        for(i in 2:N){
                # Define process with error
                mu.temp.r[i,4] <- alpha[4] * log.R[i-1,4] + beta[4]
                log.R[i,4] ~ dnorm(mu.temp.r[i,4], tau.process.r[4])

                # Observation error
                mu.r[i,4] ~ dnorm(log.R[i,4], tau.r[i,4]);

                # Calculate residuals:
                res[i,4] <- log.R[i,4] - mu.r[i,4]
                res2[i,4] <- log.R[i,4] - mu.temp.r[i,4]
```

```
}

# State regression models:
for (j in 1:3){
        for(i in 1:N){
                # Define process with error
                mu.temp.r[i,4-j] <- alpha[4-j]*log.R[i,5-j] + beta[4-j];
                log.R[i,4-j] ~ dnorm(mu.temp.r[i,4-j], tau.process.r[4-j])

                # Observation error
                mu.r[i,4-j] ~ dnorm(log.R[i,4-j], tau.r[i,4-j]);

                # Calculate residuals:
                res[i,4-j] <- log.R[i,4-j] - mu.r[i,4-j]
                res2[i,4-j] <- log.R[i,4-j] - mu.temp.r[i,4-j]
        }
}

# Convert log-abundances to true scale:
for (i in 1:N){
        for (j in 1:4){
                R[i,j] <- exp(log.R[i,j])
        }
}

# Define regression model on fishable population:
ER <- 0.35  # Target exploitation rate.
kappa ~ dnorm(0,0.0001)  # Prior on remaining regression coefficient.

# Process error prior
tau.process.rem ~ dgamma(1.0E-4, 1.0E-4) I(0.000001,)
sigma.process.rem <- 1/sqrt(tau.process.rem)
# Prior for error of missing data
sigma.rem[9] ~ dgamma(4.6, 37.8)
for (i in 1:18) {
        tau.rem[i] <- 1/pow(sigma.rem[i],2);
}
# Define initial remaining abundance according to kriged estimate
log.Rem[1] ~ dnorm(mu.rem[1], tau.rem[1])
Rem[1] <- exp(log.Rem[1])
mu.weight ~ dnorm(0, 1.0E-6) I(0.000001,)   # Mean weight prior mean.
tau.weight ~ dgamma(1.0E-4, 1.0E-4)          # Mean weight prior pecision.
sigma.weight <- 1/sqrt(tau.weight)
weight[1] ~ dnorm(mu.weight, tau.weight) I(0.000001,)
for (i in 2:18){
        weight[i] ~ dnorm(mu.weight, tau.weight) I(0.000001,)
```

```
                log.C[i] <- log(L[i] / weight[i])
                C[i] <- exp(log.C[i])

                # Define process with error
                mu.temp.rem[i] <- kappa*log.R[i-1,1]+kappa*log.Rem[i-1] -log.C[i]
                log.Rem[i] ~ dnorm(mu.temp.rem[i], tau.process.rem)
                Rem[i] <- exp(log.Rem[i])

                # Observation error
                mu.rem[i] ~ dnorm(log.Rem[i], tau.rem[i]);

                # Calculate residuals:
                Rem.res[i] <- log.Rem[i] - mu.rem[i]
                Rem.res2[i] <- log.Rem[i] - mu.temp.rem[i]
        }

        # Mean sample weight for projecting landings:
        for (i in 19:22){
                weight[i] ~ dnorm(mu.weight, tau.weight) I(0.000001,)
                C[i] <- ER * (R[i-1,1] +  Rem[i-1])
                log.C[i] <- log(C[i])
                mu.temp.rem[i] <- kappa*log.R[i-1,1]+kappa*log.Rem[i-1] -log.C[i]
                log.Rem[i] ~ dnorm(mu.temp.rem[i], tau.process.rem) I(,8)
                Rem[i] <- exp(log.Rem[i])
                L[i] <- weight[i] * C[i]
        }

        # Assess risk of exceeding target exploitation rate ER:
        for (j in 1:51){
                L.index[j] <- (j-1)*1000
                C.index[18,j] <- L.index[j] / weight[18]
        }
        for (j in 1:51){
                for (i in 19:22){
                        C.index[i,j] <- L.index[j] / weight[i]
                        ER.index[i,j] <- C.index[i,j] / (R[i-1,1] + Rem[i-1])
                        ER.delta[i,j] <- ER.index[i,j] - ER
                        ptargER[i,j] <- step(ER.delta[i,j])
                }
        }
}

# Define data:

list(
        # Number of years to predict recruitment (1988 up to 2008) of R-1s:
```

```
N = 21,
# These are kriged estimates for recruitment abundances (in millions)
# transformed to the log-scale:
mu.r = structure(
        .Data = c(0.404, NA, NA, NA,
                1.932, 2.849, NA, NA,
                3.765, 3.981, 3.517, NA,
                3.718, 4.801, 4.6, 4.242,
                4.279, 4.982, 5.285, 4.954,
                4.863, 5.148, 4.747, 5.518,
                4.521, 5.074, 4.811, 4.756,
                3.718, 5.001, 4.569, 4.223,
                NA, 4.418, 4.609, 3.329,
                3.731, NA, 3.644, 3.802,
                3.547, 3.858, NA, 3.743,
                3.566, 3.823, 3.991, NA,
                3.763, 4.561, 4.194, 4.5,
                3.873, 4.209, 4.677, 4.707,
                4.141, 4.513, 4.669, 4.907,
                4.139, 5.004, 5.079, 4.965,
                4.614, 4.967, 5.09, 5.396,
                4.241, 4.897, 4.995, 4.801,
                NA, 4.792, 4.363, 4.409,
                NA, NA, 4.001, 3.895,
                NA, NA, NA, 3.291),
        .Dim = c(21, 4)),

# These are kriged estimates for recruitment abundance standard deviations
#  (in millions) transformed to the log-scale:
sigma.r = structure(
                .Data = c(0.363, NA, NA, NA,
                        0.343, 0.245, NA, NA,
                        0.183, 0.213, 0.1591, NA,
                        0.205, 0.129, 0.0998, 0.4762,
                        0.124, 0.109, 0.0862, 0.1309,
                        0.160, 0.146, 0.1206, 0.0827,
                        0.081, 0.128, 0.1341, 0.1512,
                        0.727, 0.085, 0.1518, 0.1568,
                        NA, 0.115, 0.1115, 0.1871,
                        0.086, NA, 0.1336, 0.1206,
                        0.124, 0.11, NA, 0.1182,
                        0.09, 0.146, 0.1174, NA,
                        0.086, 0.137, 0.0806, 0.1004,
                        0.117, 0.095, 0.097, 0.0731,
                        0.081, 0.106, 0.085, 0.11,
                        0.069, 0.083, 0.0621, 0.0977,
```

```
                0.047, 0.077, 0.0845, 0.0615,
                0.052, 0.062, 0.0729, 0.0799,
                NA, 0.073, 0.0801, 0.1321,
                NA, NA, 0.0589, 0.109,
                NA, NA, NA, 0.1339),
        .Dim = c(21, 4)),

    # These are kriged estimates for remaining abundances (in millions)
    # transformed to the log-scale:
    mu.rem = c(2.734, 3.298, 3.336, 3.232, 3.845, 4.508, 4.669,
               4.559, NA, 3.616, 3.62, 3.59, 2.157, 2.969,
               2.787, 3.537, 3.299, 3.404),

    # These are kriged estimates for remaining abundance standard deviations
    #  (in millions) transformed to the log-scale:
    sigma.rem = c(0.122, 0.087, 0.099, 0.113, 0.155, 0.086,
                  0.061, 0.067, NA, 0.088, 0.097, 0.101, 0.232,
                  0.124, 0.086, 0.09, 0.087, 0.088),

    # Reported landings for zone 12 (in metric tons)
    L = c(12355, 7882, 6950, 10019, 11235, 14336, 19995,
          19944, 15978, 15413, 11136, 12682, 15046, 13819,
          21869, 16898, 26213, 32400, NA, NA, NA, NA),

    # Mean weights for each year (in grams)
    weight = c(589.4, 576.4, 517.2, 542.7, 559.3, 563.3,
               560.9, 587.7, NA, 594.2, 586.5, 541.6,
               571.7, 524.9, 524.5, 548.6, 565.4, 582.8, NA, NA,
               NA, NA),
)
```